1. Units and Measurements

Can you recall?

1. What is a unit?
2. Which units have you used in the laboratory for measuring
   (i) length (ii) mass (iii) time (iv) temperature?
3. Which system of units have you used?

1.1 Introduction:

Physics is a quantitative science, where we measure various physical quantities during experiments. In our day to day life, we need to measure a number of quantities, e.g., size of objects, volume of liquids, amount of matter, weight of vegetables or fruits, body temperature, length of cloth, etc. A measurement always involves a comparison with a standard measuring unit which is internationally accepted. For example, for measuring the mass of a given fruit we need standard mass units of 1 kg, 500 g, etc. These standards are called units. The measured quantity is expressed in terms of a number followed by a corresponding unit, e.g., the length of a wire is written as 5 m where m (metre) is the unit and 5 is the value of the length in that unit. Different quantities are measured in different units, e.g. length in metre (m), time in seconds (s), mass in kilogram (kg), etc. The standard measure of any quantity is called the unit of that quantity.

1.2 System of Units:

In our earlier standards we have come across various systems of units namely

(i) CGS: Centimetre Gram Second system

(ii) MKS: Metre Kilogram Second system

(iii) FPS: Foot Pound Second system.

(iv) SI: System International

The first three systems namely CGS, MKS and FPS were used extensively till recently. In 1971, the 14th International general conference on weights and measures recommended the use of 'International system' of units. This international system of units is called the SI units. As the SI units use decimal system, conversion within the system is very simple and convenient.

1.2.1 Fundamental Quantities and Units:

The physical quantities which do not depend on any other physical quantities for their measurements are known as fundamental quantities. There are seven fundamental quantities: length, mass, time, temperature, electric current, luminous intensity and amount of substance.

Fundamental units: The units used to measure fundamental quantities are called fundamental units. The fundamental quantities, their units and symbols are shown in the Table 1.1.

Table 1.1: Fundamental Quantities with their SI Units and Symbols

Fundamental quantity	SI units	Symbol
1) Length	metre	m
2) Mass	kilogram	kg
3) Time	second	s
4) Temperature	kelvin	K
5) Electric current	ampere	A
6) Luminous Intensity	candela	cd
7) Amount of substance	mole	mol

1.2.2 Derived Quantities and Units:

In physics, we come across a large number of quantities like speed, momentum, resistance, conductivity, etc. which depend on some or all of the seven fundamental quantities and can be expressed in terms of these quantities. These are called derived quantities and their units, which can be expressed in terms of the fundamental units, are called derived units.

For example,

SI unit of velocity

$$= \frac{\text{Unit of displacement}}{\text{Unit of time}} = \frac{m}{s} = m\,s^{-1}$$

Unit of momentum = (Unit of mass)×(Unit of velocity)

$= kg\ m/s = kg\ m\ s^{-1}$

The above two units are derived units.

Supplementary units : Besides, the seven fundamental or basic units, there are two more units called supplementary units: (i) Plane angle dθ and (ii) Solid angle dΩ

(i) Plane angle (dθ) : This is the ratio of the length of an arc of a circle to the radius of the circle as shown in Fig. 1.1 (a). Thus dθ = ds/r is the angle subtended by the arc at the centre of the circle. It is measured in radian (rad). An angle θ in radian is denoted as θ^c.

(ii) Solid angle (dΩ) : This is the 3-dimensional analogue of dθ and is defined as the area of a portion of surface of a sphere to the square of radius of the sphere. Thus $d\Omega = dA/r^2$ is the solid angle subtended by area ds at O as shown in Fig. 1.1 (b). It is measured in steradians (sr). A sphere of radius r has surface area $4\pi r^2$. Thus, the solid angle subtended by the entire sphere at its centre is $\Omega = 4\pi r^2/r^2 = 4\pi$ sr.

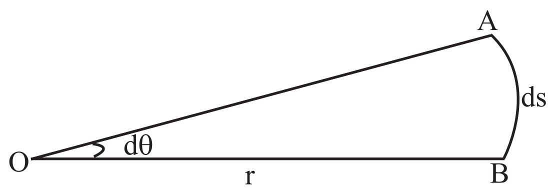


Fig 1.1 (a): Plane angle dθ.

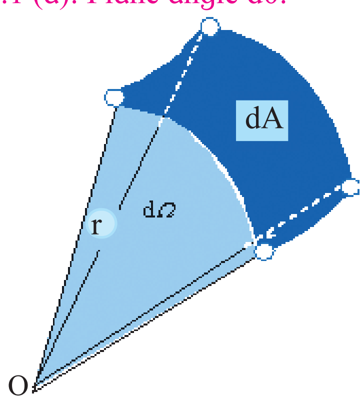


Fig 1.1 (b): Solid angle dΩ.

Example 1.1: What is the solid angle subtended by the moon at any point of the Earth, given the diameter of the moon is 3474 km and its distance from the Earth 3.84×10^8 m.

Solution: Solid angle subtended by the moon at the Earth

$$= \frac{\text{Area of the disc of the moon}}{(\text{moon - earth distance})^2}$$

$$= \frac{\pi \times (1.737\times10^3)^2}{(3.84\times10^5)^2}$$

$$= 6.425\times10^{-5}\ sr$$

Do you know ?

The relation between radian and degree is

π radians $= \pi^c = 180°$

$$\therefore\ 1\ \text{radian} = \frac{180}{\pi} = \frac{180}{3.1415} = 57.297°$$

$$\text{Similarly } 1° = \frac{\pi}{180} = \frac{3.1415}{180} = 1.745\times10^{-2}\ \text{rad}$$

$$1° = 60',\ 1' = 2.91\times10^{-4}\ \text{rad}$$

$$\text{and } 1' = 60'',\ 1'' = 4.847\times10^{-6}\ \text{rad}$$

1.2.3 Conventions for the use of SI Units:

(1) Unit of every physical quantity should be represented by its symbol.

(2) Full name of a unit always starts with smaller letter even if the name is after a person, e.g., 1 newton, 1 joule, etc. But symbol for unit named after a person should be in capital letter, e.g., N after scientist Newton, J after scientist Joule, etc.

(3) Symbols for units do not take plural form for example, force of 20 N and not 20 newtons or not 20 Ns.

(4) Symbols for units do not contain any full stops at the end of recommended letter, e.g., 25 kg and not 25 kg..

(5) The units of physical quantities in numerator and denominator should be written as one ratio for example the SI unit of acceleration is m/s^2 or $m\ s^{-2}$ but not m/s/s.

(6) Use of combination of units and symbols for units is avoided when physical quantity is expressed by combination of two. e.g., The unit J/kg K is correct while joule/kg K is not correct.

(7) A prefix symbol is used before the symbol of the unit.

Thus prefix symbol and units symbol constitute a new symbol for the unit which can be raised to a positive or negative power of 10.

1ms = 1 millisecond = 10^{-3}s
1µs = 1 microsecond = 10^{-6}s
1ns = 1 nanosecond = 10^{-9}s
Use of double prefixes is avoided when single prefix is available
10^{-6}s =1µs and not 1mms.
10^{-9}s = 1ns and not 1mµs

(8) Space or hyphen must be introduced while indicating multiplication of two units e.g., m/s should be written as m s^{-1} or m-s^{-1} and Not as ms^{-1} (because ms will be read as millisecond).

1.3 Measurement of Length:

One fundamental quantity which we have discussed earlier is length. To measure the length or distance the SI unit used is metre (m). In 1960, a standard for the metre based on the wavelength of orange-red light emitted by atoms of krypton was adopted. By 1983 a more precise measurement was developed. It says that a metre is the length of the path travelled by light in vacuum during a time interval of 1/299792458 second. This was possible as by that time the speed of light in vacuum could be measured precisely as c = 299792458 m/s

Some typical distances/lengths are given in Table 1.2.

Table 1.2: Some Useful Distances

Measurement	Length in metre
Distance to Andromeda Galaxy (from Earth)	2×10^{22} m
Distance to nearest star (after Sun) Proxima Centuari (from Earth)	4×10^{16} m
Distance to Pluto (from Earth)	6×10^{12} m
Average Radius of Earth	6×10^{6} m
Height of Mount Everest	9×10^{3} m
Thickness of this paper	1×10^{-4} m
Length of a typical virus	1×10^{-8} m
Radius of hydrogen atom	5×10^{-11} m
Radius of proton	1×10^{-15} m

1.3.1 Measurements of Large Distance: Parallax method

Large distance, such as the distance of a planet or a star from the Earth, cannot be measured directly with a metre scale, so a parallax method is used for it.

Let us do a simple experiment to understand what is parallax.

Hold your hand in front of you and look at it with your left eye closed and then with your right eye closed. You will find that your hand appears to move against the background. This effect is called parallax. Parallax is defined as the apparent change in position of an object due to a change in the position of the observer. By measuring the parallax angle (θ) and knowing the distance between the eyes E_1E_2 as shown in Fig. 1.2, we can determine the distance of the object from us, i.e., OP as E_1E_2/θ.

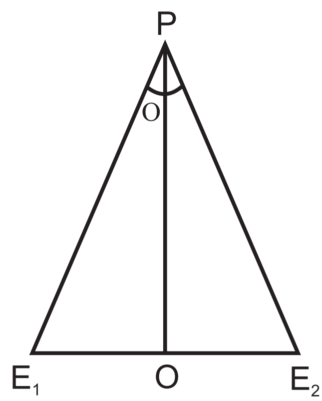


Fig.1.2: Parallax method for determining distance.

As the distances of planets from the Earth are very large, we can not use two eyes method as discussed here. In order to make simultaneous observations of an astronomical object, we select two distant points on the Earth.

Consider two positions A and B on the surface of Earth, separated by a straight line at distance b as shown in Fig. 1.3. Two observers at these two points observe a distant planet S simultaneously. We measure the angle ∠ASB between the two directions along which the planet is viewed at these two points. This angle, represented by symbol θ, is the parallax angle.

As the planet is far away, i.e., the distance of the planet from the Earth is very large in comparison to b, $b/D << 1$ and, therefore, θ is very small.

We can thus consider AB as the arc of length b of the circle and D its radius.

AB = b and AS = BS = D and $\theta \cong AB/D$, where θ is in radian

$$D = b / \theta$$

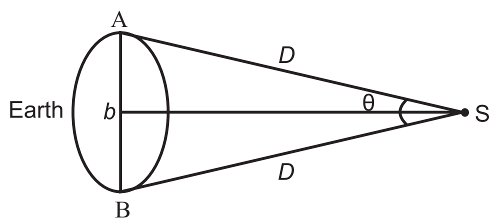


Fig.1.3: Measurement of distances of planets

1.3.2 Measurement of Distance to Stars:

Sun is the star which is closest to the Earth. The next closest star is at a distance of 4.29 light years. The parallax measured from two most distance points on the Earth for stars will be too small to be measured and for this purpose we measure the parallax between two farthest points (i.e. 2 AU apart, see box below) along the orbit of the Earth around the Sun (see figure in example 1.2 below).

1.3.3 Measurement of the Size of a Planet or a Star:

If d is the diameter of a planet, the angle subtended by it at any single point on the Earth is called angular diameter of the planet. Let α be the angle between the two directions when two diametrically opposite points of the planet are viewed through a telescope as shown in Fig. 1.4. As the distance D of the planet is large (assuming it has been already measured), we can calculate the diameter of the planet as

$$\alpha = \frac{d}{D}$$

$$\therefore d = \alpha D \qquad \text{--- (1.2)}$$

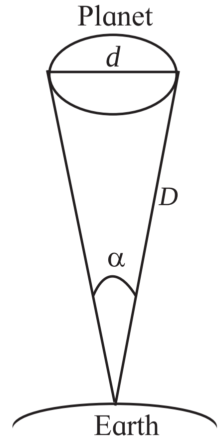


Fig. 1.4: Measurement of size of a planet

1.3.4 Measurement of Very Small Distances:

When we intend to measure the size of the atoms and molecules, the conventional apparatus like Vernier calliper or screw guage will not be useful. Therefore, we use electron microscope or tunnelling electron microscope to measure the size of atoms.

Do you know ?

For measuring large distances, astronomers use the following units.

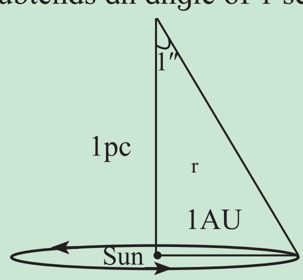
1 astronomical unit, (AU) = 1.496×10^{11}m

1 light year = 9.46×10^{15}m

1 parsec (pc) = 3.08×10^{16}m ≅ 3.26 light years

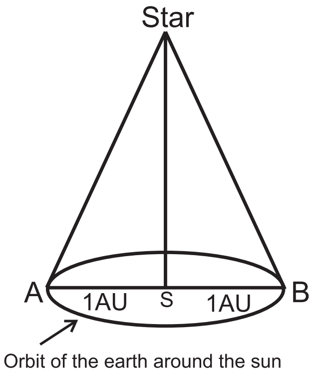
A light year is the distance travelled by light in one year. The astronomical unit (AU) is the mean distance between the centre of the Earth and the centre of the Sun.

A parsec (pc) is the distance from where 1AU subtends an angle of 1 second of arc.



$$r = \frac{1AU}{(1")^c} = \frac{1.496\times10^{11}}{4.847\times10^{-6}} = 3.08\times10^{16}\text{ m}$$

Example 1.2: A star is 5.5 light years away from the Earth. How much parallax in arcsec will it subtend when viewed from two opposite points along the orbit of the Earth?



Solution: Two opposite points A and B along the orbit of the Earth are 2 AU apart. The angle subtended by AB at the position of the star is = AB/distance of the star from the Earth

$$= \frac{2AU}{5.5\,ly} = \frac{2 \times 1.496 \times 10^{11}\,m}{5.5 \times 9.46 \times 10^{15}\,m} = 5.75 \times 10^{-6}\,rad$$

$$= 5.75 \times 10^{-6} \times 57.297 \times 60 \times 60 \text{ arcsec}$$

$$= 1.186 \text{ arcsec}$$

Do you know ?

Small distances are measured in units of (i) fermi = 1F = 10^{-15} m in SI system. Thus, 1F is one femtometre (fm) (ii) Angstrom = 1 A^0 = 10^{-10} m

For measuring sizes using a microscope we need to select the wavelength of light to be used in the microscope such that it is smaller than the size of the object to be measured. Thus visible light (wavelength from 4000 A^0 to 7000 A^0) can measure sizes upto about 4000 A^0. If we want to measure even smaller sizes we need to use even smaller wavelength and so the use of electron microscope is necessary. As you will study in the XIIth standard, each material particle corresponds to a wave. The approximate wavelength of the electrons in an electron microscope is about 0.6 A^0 so that one can measure atomic sizes ≈ 1 A^0 using this microscope.

Example 1.3: The moon is at a distance of 3.84×10^8 m from the Earth. If viewed from two diametrically opposite points on the Earth, the angle subtended at the moon is 1° 54'. What is the diameter of the Earth?

Solution: Angle subtended

$\theta = 1° \, 54' = 114' = 114\times2.91\times10^{-4}$ rad

$= 3.317\times10^{-2}$ rad

Diameter of the Earth = θ × distance to the moon from the Earth

$= 3.317\times10^{-2}\times3.84\times10^8$ m

$= 1.274\times10^7$ m

1.4 Measurement of Mass:

Since 1889, a kilogram was the mass of a shiny piece of platinum-iridium alloy kept in a special glass case at the International Bureau of weights and measures. This definition of mass has been modified on 20th May 2019, the reason being that the carefully kept platinum-iridium piece is seen to pick up micro particles of dirt and is also affected by the atmosphere causing its mass to change. The new measure of kilogram is defined in terms of magnitude of electric current. We know that electric current can be used to make an electromagnet. An electromagnet attracts magnetic materials and is thus used in research and in industrial applications such as cranes to lift heavy pieces of iron/steel. Thus the kilogram mass can be described in terms of the amount of current which has to be passed through an electromagnet so that it can pull down one side of an extremely sensitive balance to balance the other side which holds one standard kg mass.

While dealing with mass of atoms and molecules, kg is an inconvenient unit. Therefore, their mass is measured in atomic mass unit. It will be easy to compare mass of any atom in terms of mass of some standard atom which has been decided internationally to be C^{12} atom. The (1/12)th mass of an unexcited atom of C^{12} is called atomic mass unit (amu).

1 amu = 1.6605402×10^{-27} kg with an uncertainty of 10 in the last two decimal places.

1.5 Measurement of Time:

The SI unit of time is the second (s). For many years, duration of one mean Solar day was considered as reference. A mean Solar day is the average time interval from one noon to the next noon. Average duration of a day is taken as 24 hours. One hour is of 60 minutes and each minute is of 60 seconds. Thus a mean Solar day = 24 hours = $24 \times 60 \times 60$ = 86400 s. Accordingly a second was defined as 1/86400 of a mean Solar day.

It was later observed that the length of a Solar day varies gradually due to the gradual slowing down of the Earth's rotation. Hence, to get more standard and nonvarying (constant) unit for measurement of time, a cesium atomic clock is used. It is based on periodic vibrations produced in cesium atom. In cesium atomic clock, a second is taken as the time needed for 9,192,631,770 vibrations of the radiation (wave) emitted during a transition between two hyperfine states of Cs^{133} atom.

Do you know ?

Why is only carbon used and not any other element for defining atomic mass unit? Carbon 12 (C^{12}) is the most abundant isotope of carbon and the most stable one. Around 98% of the available carbon is C^{12} isotope.

Earlier, oxygen and hydrogen were used as the standard atoms. But various isotopes of oxygen and hydrogen are present in higher proportion and therefore it is more accurate to use C^{12}.

1.6 Dimensions and Dimensional Analysis:

As mentioned earlier, a derived physical quantity can be expressed in terms of some combination of seven basic or fundamental quantities. For convenience, the basic quantities are represented by symbols as 'L' for length, 'M' for mass, 'T' for time, 'K' For temperature, 'I' for current, 'C' for luminous intensity and 'mol' for amount of mass.

The dimensions of a physical quantity are the powers to which the concerned fundamental units must be raised in order to obtain the unit of the given physical quantity.

When we represent any derived quantity with appropriate powers of symbols of the fundamental quantities, then such an expression is called dimensional formula. This dimensional formula is expressed by square bracket and no comma is written in between any of the symbols.

Illustration:

(i) Dimensional formula of velocity

$$\text{Velocity} = \frac{\text{displacment}}{\text{time}}$$

$$\text{Dimensions of velocity} = \frac{[L]}{[T]} = [L^{1}M^{0}T^{-1}]$$

ii) Dimensional formula of velocity gradient

$$\text{velocity gradient} = \frac{\text{velocity}}{\text{distance}}$$

Dimensions of velocity gradient

$$= \frac{[LT^{-1}]}{[L]} = [L^{0}M^{0}T^{-1}]$$

iii) Dimensional formula for charge.

charge = current $\times$ time

Dimensions of charge = [I] [T] = $[L^{0}M^{0}T^{1}I^{1}]$

Table 1.3: Some Common Physical Quantities their, SI Units and Dimensions

S. No	Physical quantity	Formula	SI unit	Dimensional formula
1	Density	$\rho = M/V$	kilogram per cubic metre (kg/m^3)	$[L^{-3}M^{1}T^{\circ}]$
2	Acceleration	$a = v/t$	metre per second square (m/s^2)	$[L^{1}M^{\circ}T^{-2}]$
3	Momentum	$P = mv$	kilogram metre per second (kg m/s)	$[L^{1}M^{1}T^{-1}]$
4	Force	$F = ma$	kilogram metre per second square (kg m/s^2) or newton (N)	$[L^{1}M^{1}T^{-2}]$
5	Impulse	$J = F.\,t$	newton second (Ns)	$[L^{1}M^{1}T^{-1}]$
6	Work	$W = F.s$	joule (J)	$[L^{2}M^{1}T^{-2}]$
7	Kinetic Energy	$KE = 1/2\, mv^2$	joule (J)	$[L^{2}M^{1}T^{-2}]$
8	Pressure	$P = F/A$	kilogram per metre second square (kg/ms^2)	$[L^{-1}M^{1}T^{-2}]$

Table 1.3 gives the dimensions of various physical quantities commonly used in mechanics.

1.6.1 Uses of Dimensional Analysis:

(i) To check the correctness of physical equations: In any equation relating different physical quantities, if the dimensions of all the terms on both the sides are the same then that equation is said to be dimensionally correct. This is called the principle of homogeneity of dimensions. Consider the first equation of motion.

$v = u + at$

Dimension of L.H.S = $[v] = [LT^{-1}]$

$[u] = [LT^{-1}]$

$[at] = [LT^{-2}]\,[T] = [LT^{-1}]$

Dimension of R.H.S $= [LT^{-1}] + [LT^{-1}]$

[L.H.S] = [R.H.S]

As the dimensions of L.H.S and R.H.S are the same, the given equation is dimensionally correct.

(ii) To establish the relationship between related physical quantities: The period T of oscillation of a simple pendulum depends on length l and acceleration due to gravity g. Let us derive the relation between T, l, g :

Suppose $T \propto l^a$

and $T \propto g^b$

$\therefore T \propto l^a g^b$

$T = k\, l^a g^b$,

where k is constant of proportionality and it is a dimensionless quantity and a and b are rational numbers. Equating dimensions on both sides,

$[M^0L^0T^1] = k\,[L^1]^a\,[LT^{-2}]^b$

$= k\,[L^{a+b}T^{-2b}]$

$[L^0T^1] = k\,[L^{a+b}T^{-2b}]$

Comparing the dimensions of the corresponding quantities on both the sides we get

$a + b = 0$

$\therefore a = -b$

and

$-2b = 1$

$\therefore b = -1/2$

$\therefore a = -b = -(-1/2)$

$\therefore a = 1/2$

$\therefore T = k\, l^{1/2}\, g^{-1/2}$

$\therefore T = k\sqrt{l/g}$

The value of k is determined experimentally and is found to be 2π

$\therefore T = 2\pi\sqrt{l/g}$

(iii) To find the conversion factor between the units of the same physical quantity in two different systems of units: Let us use dimensional analysis to determine the conversion factor between joule (SI unit of work) and erg (CGS unit of work).

Let 1 J = x erg

Dimensional formula for work is $[M^1L^2T^{-2}]$

Substituting in the above equation, we can write

$$[M_1^1L_1^2T_1^{-2}] = x\,[M_2^1L_2^2T_2^{-2}]$$

$$x = \frac{[M_1^1L_1^2T_1^{-2}]}{[M_2^1L_2^2T_2^{-2}]}$$

$$\text{or, } x = \left(\frac{M_1}{M_2}\right)^1 \left(\frac{L_1}{L_2}\right)^2 \left(\frac{T_1}{T_2}\right)^{-2}$$

where suffix 1 indicates SI units and 2 indicates CGS units.

In SI units, L, M, T are expressed in m, kg and s and in CGS system L, M, T are represented in cm, g and s respectively.

$$\therefore x = \left(\frac{kg}{g}\right)^1 \left(\frac{m}{cm}\right)^2 \left(\frac{s}{s}\right)^{-2}$$

$$\text{or } x = \left(10^3\frac{g}{g}\right)^1 \left((100)\frac{cm}{cm}\right)^2 (1)^{-2}$$

$$\therefore x = (10^3)(10^4) = 10^7$$

$$\therefore 1\text{ joule} = 10^7\text{ erg}$$

Example 1.4: A calorie is a unit of heat and it equals 4.2 J, where 1 J = kg m^2 s^{-2}. A distant civilisation employs a system of units in which the units of mass, length and time are α kg, β m and γ s. Also J' is their unit of energy. What will be the magnitude of calorie in their units?

Solution: Let us write the new units as A, B and C for mass, length and time respectively. We are given

1 A = α kg

1 B = β m

1 C = γ s

1 cal = 4.2 J = 4.2 kg m^2 s^{-2}

$$= 4.2\left(\frac{A}{\alpha}\right)\left(\frac{B}{\beta}\right)^2\left(\frac{C}{\gamma}\right)^{-2}$$

$$= \frac{4.2\,\gamma^2}{\alpha\beta^2}\ AB^2\,C^{-2}$$

$$= \frac{4.2\,\gamma^2}{\alpha\beta^2}\ J'$$

Thus in the new units, 1 calorie is $= \frac{4.2\gamma^2}{\alpha\beta^2}$ J'

1.6.2 Limitations of Dimensional Analysis:

1) The value of dimensionless constant can be obtained with the help of experiments only.

2) Dimensional analysis can not be used to derive relations involving trigonometric, exponential, and logarithmic functions as these quantities are dimensionless.

3) This method is not useful if constant of proportionality is not a dimensionless quantity.

   Illustration : Gravitational force between two point masses is directly proportional to product of the two masses and inversely proportional to square of the distance between the two

   $$\therefore F \propto \frac{m_1 m_2}{r^2}$$

   $$\text{Let } F = G\frac{m_1 m_2}{r^2}$$

   The constant of proportionality 'G' is NOT dimensionless. Thus earlier method will not work.

4) If the correct equation contains some more terms of the same dimension, it is not possible to know about their presence using dimensional equation. For example, with standard symbols, the equation $S = \frac{1}{2}at^2$ is dimensionally correct. However, the complete equation is $S = ut + \frac{1}{2}at^2$

1.7 Accuracy, Precision and Uncertainty in Measurement:

Physics is a science based on observations and experiments. Observations of various physical quantities are made during an experiment. For example, during the atmospheric study we measure atmospheric pressure, wind velocity, humidity, etc. All the measurements may be accurate, meaning that the measured values are the same as the true values. Accuracy is how close a measurement is to the actual value of that quantity. These measurements may be precise, meaning that multiple measurements give nearly identical values (i.e., reproducible results). In actual measurements, an observation may be both accurate and precise or neither accurate nor precise. The goal of the observer should be to get accurate as well as precise measurements.

Possible uncertainties in an observation may arise due to following reasons:

1) Quality of instrument used.

2) Skill of the person doing the experiment.

3) The method used for measurement.

4) External or internal factors affecting the result of the experiment.

Can you tell?

If ten students are asked to measure the length of a piece of cloth up to a mm, using a metre scale, do you think their answers will be identical? Give reasons.

1.8 Errors in Measurements:

Faulty measurements of physical quantity can lead to errors. The errors are broadly divided into the following two categories :

a) Systematic errors : Systematic errors are errors that are not determined by chance but are introduced by an inaccuracy (involving either the observation or measurement process) inherent to the system. Sources of systematic error may be due to imperfect calibration of the instrument, and sometimes imperfect method of observation.

Each of these errors tends to be in one direction, either positive or negative. The sources of systematic errors are as follows:

(i) **Instrumental error:** This type of error arises due to defective calibration of an instrument, for example an incorrect zeroing of an instrument will lead to such kind of error ('zero' of a thermometer not graduated at proper place, the pointer of weighting balance in the laboratory already indicating some value instead of showing zero when no load is kept on it, an ammeter showing a current of 0.5 amp even when not connected in circuit, etc).

(ii) **Error due to imperfection in experimental technique:** This is an error due to defective setting of an instrument. For example the measured volume of a liquid in a graduated tube will be inaccurate if the tube is not held vertical.

(iii) **Personal error:** Such errors are introduced due to fault of the observer. Bias of the observer, carelessness in taking observations etc. could result in such errors. For example, while measuring the length of an object with a ruler, it is necessary to look at the ruler from directly above. If the observer looks at it from an angle, the measured length will be wrong due to parallax.

Systematic errors can be minimized by using correct instrument, following proper experimental procedure and removing personal error.

b) Random errors: These are the errors which are introduced even after following all the procedures to minimize systematic errors. These type of errors may be positive or negative. These errors can not be eliminated completely but we can minimize them by repeated observations and then taking their mean (average). Random errors occur due to variation in conditions in which experiment is performed. For example, the temperature may change during the course of an experiment, pressure of any gas used in the experiment may change, or the voltage of the power supply may change randomly, etc.

1.8.1 Estimation of error:

Suppose the readings recorded repeatedly for a physical quantity during a measurement are

$a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_n$.

Arithmetic mean a_{mean} is given by

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots + a_n}{n}$$

$$a_{mean} = \frac{1}{n}\sum_{i=1}^{n} a_i \qquad \text{--- (1.3)}$$

This is the most probable value of the quantity. The magnitude of the difference between mean value and each individual value is called **absolute error** in the observations.

Thus for 'a_1', the absolute error Δa_1 is given by

$$\Delta a_1 = |a_{mean} - a_1|,$$

for a_2,

$$\Delta a_2 = |a_{mean} - a_2|$$

and so for a_n it will be

$$\Delta a_n = |a_{mean} - a_n|$$

The arithmetic mean of all the absolute errors is called **mean absolute error** in the measurement of the physical quantity.

$$\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 + \ldots\ldots + \Delta a_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \Delta a_i \qquad \text{--- (1.4)}$$

The measured value of the physical quantity *a* can then be represented by

$a = a_{mean} \pm \Delta a_{mean}$ which tells us that the actual value of 'a' could be between $a_{mean} - \Delta a_{mean}$ and $a_{mean} + \Delta a_{mean}$. The ratio of mean absolute error to its arithmetic mean value is called **relative error**.

$$\text{Relative error} = \frac{\Delta a_{mean}}{a_{mean}} \qquad \text{--- (1.5)}$$

When relative error is represented as percentage it is called **percentage error**.

$$\text{Percentage error} = \frac{\Delta a_{mean}}{a_{mean}} \times 100 \qquad \text{--- (1.6)}$$

Activity :

Perform an experiment using a Vernier callipers of least count 0.01cm to measure the external diameter of a hollow cylinder. Take 3 readings at different position on the cylinder and find (i) the mean diameter (ii) the absolute mean error and (iii) the percentage error in the measurement of diameter.

Example 1.5: The radius of a sphere measured repeatedly yields values 5.63 m, 5.54 m, 5.44 m, 5.40 m and 5.35 m. Determine the most probable value of radius and the mean absolute, relative and percentage errors.

Solution: Most probable value of radius is its arithmetic mean

$$= \frac{5.63 + 5.54 + 5.44 + 5.40 + 5.35}{5} \text{ m}$$

$$= 5.472 \text{ m}$$

Mean absolute error

$$= \frac{1}{5}\left\{\begin{array}{l} |5.63 - 5.472| + |5.54 - 5.472| \\ + |5.44 - 5.472| + |5.40 - 5.472| \\ + |5.35 - 5.472| \end{array}\right\} \text{m}$$

$$= \frac{0.452}{5} = 0.0904 \text{ m}$$

$$\text{Relative error} = \frac{0.0904}{5.472} = 0.017$$

$$\% \text{ error} = 1.7\%$$

1.8.2 Combination of errors:

When we do an experiment and measure various physical quantities associated with the experiment, we must know how the errors from individual measurement combine to give errors in the final result. For example, in the measurement of the resistance of a conductor using Ohms law, there will be an error in the measurement of potential difference and that of current. It is important to study how these errors combine to give the error in the measurement of resistance.

a) Errors in sum and in difference:

Suppose two physical quantities A and B have measured values A ± ΔA and B ± ΔB, respectively, where ΔA and ΔB are their mean absolute errors. We wish to find the absolute error ΔZ in their sum.

$$Z = A + B$$

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$

$$= (A+B) \pm \Delta A \pm \Delta B$$

$$\pm \Delta Z = \pm \Delta A \pm \Delta B,$$

For difference, i.e., if Z = A-B,

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$

$$= (A-B) \pm \Delta A \pm \Delta B$$

$$\pm \Delta Z = \pm \Delta A \pm \Delta B,$$

There are four possible values for ΔZ, namely (+ ΔA - ΔB), (+ΔA+ΔB), (-ΔA-ΔB), (-ΔA+ΔB). Hence maximum value of absolute error is ΔZ = ΔA+ΔB in both the cases.

When two quantities are added or subtracted, the absolute error in the final result is the *sum* of the absolute errors in the individual quantities.

b) Errors in product and in division:

Suppose Z=AB and measured values of A and B are (A ± ΔA) and (B ± ΔB) Then

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$$

$$= AB \pm A\Delta B \pm B\Delta A \pm \Delta A\Delta B$$

Dividing L.H.S by Z and R.H.S. by AB we get

$$\left(1 \pm \frac{\Delta z}{z}\right) = \left(1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right)\right)$$

Since ΔA/A and ΔB/B are very small we shall neglect their product. Hence maximum *relative* error in Z is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B} \qquad \text{--- (1.7)}$$

This formula also applies to the division of two quantities.

Thus, when two quantities are multiplied or divided, the maximum relative error in the result is the sum of *relative* errors in each quantity.

c) Errors due to the power (index) of measured quantity:

Suppose

$$Z = A^3 = A.A.A$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta A}{A} + \frac{\Delta A}{A}$$

from the multiplication rule above.

Hence the relative error in Z $=A^3$ is three times the relative error in A. So if $Z = A^n$

$$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A} \quad \text{--- (1.8)}$$

In general, if $Z = \frac{A^p B^q}{C^r}$

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C} \quad \text{--- (1.9)}$$

The quantity in the formula which has large power is responsible for maximum error.

Example 1.6: In an experiment to determine the volume of an object, mass and density are recorded as $m = (5 \pm 0.15)$ kg and $\rho = (5 \pm 0.2)$ kg m^{-3} respectively. Calculate percentage error in the measurement of volume.

Solution : We know,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\therefore \text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{M}{\rho}$$

$$\text{Percentage error in volume} = \left(\frac{\Delta m}{m} + \frac{\Delta \rho}{\rho}\right) \times 100$$

$$= \left(\frac{0.15}{5} + \frac{0.2}{5}\right) \times 100$$

$$= (0.03 + 0.04) \times 100$$

$$= (0.07) \times 100 = 7\%$$

Example 1.7: The acceleration due to gravity is determined by using a simple pendulum of length $l = (100 \pm 0.1)$ cm. If its time period is $T = (2 \pm 0.01)$ s, find the maximum percentage error in the measurement of g.

Solution: The time period of oscillation of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring both sides

$$T^2 = 4\pi^2 l / g$$

$$\therefore g = 4\pi^2 \frac{l}{T^2}$$

Now $\Delta l = 0.1, l = 100$ cm, $\Delta T = 0.01$s, $T = 2$ s

$$\text{Percentage error} = \frac{\Delta g \times 100}{g}$$

$$= \left(\frac{\Delta l}{l} + \frac{2\Delta T}{T}\right) \times 100$$

$$= \left(\frac{0.1}{100} + \frac{2 \times 0.01}{2}\right) \times 100$$

$$= (0.001 + 0.01) \times 100 = 1.1$$

Percentage error in measurement of g is 1.1%

1.9 Significant Figures:

In the previous sections, we have studied various types of errors, their origins and the ways to minimize them. Our accuracy is limited to the least count of the instrument used during the measurement. Least count is the smallest measurement that can be made using the given instrument. For example with the usual metre scale, one can measure 0.1 cm as the least value. Hence its least count is 0.1cm.

Suppose we measure the length of a metal rod using a metre scale of least count 0.1cm. The measurement is done three times and the readings are 15.4, 15.4, and 15.5 cm. The most probable length which is the arithmetic mean as per our earlier discussion is 15.43. Out of this we are certain about the digits 1 and 5 but are not certain about the last 2 digits because of the least count limitation.

The number of digits in a measurement about which we are certain, plus one additional digit, the first one about which we are not certain is known as significant figures or significant digits.

Thus in above example, we have 3 significant digits 1, 5 and 4.

The larger the number of significant figures obtained in a measurement, the greater is the accuracy of the measurement. If one uses the instrument of smaller least count, the number of significant digits increases.

Rules for determining significant figures

1) All the nonzero digits are significant, for example if the volume of an object is 178.43 cm^3, there are five significant digits which are 1,7,8,4 and 3.
2) All the zeros between two nonzero digits are significant, eg., m = 165.02 g has 5 significant digits.
3) If the number is less than 1, the zero/zeroes on the right of the decimal point and to the left of the first nonzero digit are not significant e.g. in $\underline{0}.\underline{00}1405$, the underlined zeros are not significant. Thus the above number has four significant digits.
4) The zeros on the right hand side of the last nonzero number are significant (but for this, the number must be written with a decimal point), e.g. 1.500 or 0.01500 have both 4 significant figures each.

On the contrary, if a measurement yields length L given as

L = 125 m = 12500 cm = 125000 mm, it has only three significant digits.

To avoid the ambiguities in determining the number of significant figures, it is necessary to report every measurement in scientific notation (i.e., in powers of 10) i.e., by using the concept of order of magnitude.

The magnitude of any physical quantity can be expressed as $A\times10^n$ where 'A' is a number such that $0.5 \le A<5$ and 'n' is an integer called the **order of magnitude**.

(i) radius of Earth = 6400 km

$= 0.64\times10^7$m

The order of magnitude is 7 and the number of significant figures are 2.

(ii) Magnitude of the charge on electron *e* $= 1.6\times10^{-19}$ C

Here the order of magnitude is -19 and the number of significant digits are 2.

Internet my friend

1. videolectures.net/mit801f99_lewin_lec01/
2. hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

Definitions of SI Units

Till May 20, 2019 the *kilogram* did not have a definition; it was mass of the prototype cylinder kept under controlled conditions of temperature and pressure at the SI museum at Paris. A rigorous and meticulous experimentation has shown that the mass of the *standard* prototype for the *kilogram* has changed in the course of time. This shows the acute necessity for standardisation of units. The new definitions aim to improve the SI without changing the size of any units, thus ensuring continuity with existing measurements. In November 2018, the 26th General Conference on Weights and Measures (CGPM) unanimously approved these changes, which the International Committee for Weights and Measures (CIPM) had proposed earlier that year. These definitions came in force from 20 May 2019.

(i) As per new SI units, each of the seven fundamental units (metre, kilogram, etc.) uses ***one*** of the following 7 constants which are proposed to be having ***exact values*** as given below:

The Planck constant,
$h = 6.62607015 \times 10^{-34}$ joule-second (J s ***or*** kg m^2 s^{-1}).

The elementary charge,
$e = 1.602176634 \times 10^{-19}$ coulomb (C ***or*** A s).

The Boltzmann constant,
$k = 1.380649 \times 10^{-23}$ joule per kelvin ($J\ K^{-1}$ ***or*** kg m^2 s^{-2} K^{-1}).

The Avogadro constant (number),
$N_A = 6.02214076 \times 10^{23}$ reciprocal mole (mol^{-1}).

The speed of light in vacuum,
$c = 299792458$ metre per second ($m\ s^{-1}$).

The ground state hyperfine structure transition frequency of Caesium-133 atom,
$\Delta\nu_{Cs} = 9192631770$ hertz (Hz ***or*** s^{-1}).

The luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd} = 683 lumen per watt ($lm \cdot W^{-1}$) = 683 cd sr s^3 kg^{-1} m^{-2}, where *sr* is steradian; the SI unit of solid angle.

(ii) Definitions of the units *second* and *mole* are based only upon their respective constants, for example (a) the *second* uses ground state hyperfine structure transition frequency of Caesium-133 atom to be exactly 9192631770 hertz. Thus, the *second* is defined as 9192631770 periods of that transition, (b) the *mole* uses Avogadro's number to be $N_A = 6.02214076 \times 10^{23}$. Thus, one *mole* is that amount of substance which contains exactly $6.02214076 \times 10^{23}$ molecules.

(iii) Definitions of all the other fundamental units use one constant each and at least one other fundamental unit, for example, the *metre* makes use of speed of light in vacuum as a constant and *second* as fundamental unit. Thus, *metre* is defined as the distance traveled by the light in vacuum in exactly 1/299792458 *second*.

(iv) The figures show the dependency of various units upon their respective constants and other units (wherever used). The arrows arriving at that unit refer to the constant and the fundamental unit (or units, wherever used) for defining that unit. The arrows going away from a unit indicate other units which use this unit for their definition.

For example, as described above, in fig (a) i) the arrows directed to *metre* are from *second* and *c*. The arrows going away from the *metre* indicate that the *metre* is used in defining the *kilogram* the *candela* and the *kelvin*, (ii) the newly defined unit *kilogram* uses Planck constant, the *metre* and the *second*, while the *kilogram* itself is used in defining the *kelvin* and the *candela*. This definition relates the *kilogram* to the equivalent mass of the energy of a photon given its frequency, via the Planck constant.

Figure (a) refers to new definitions while the figure (b) refers to the corresponding definitions before 20 May 2019. Interested students may compare them to know which definitions are modified and how.

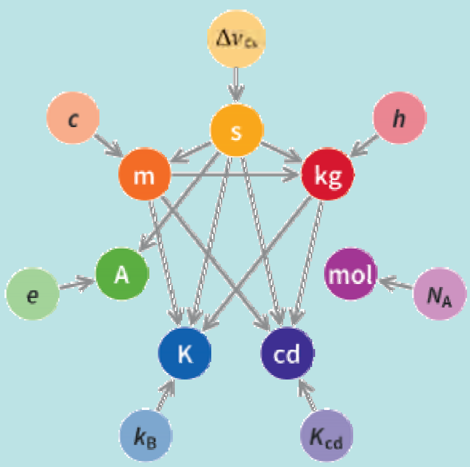


Fig (a) New SI

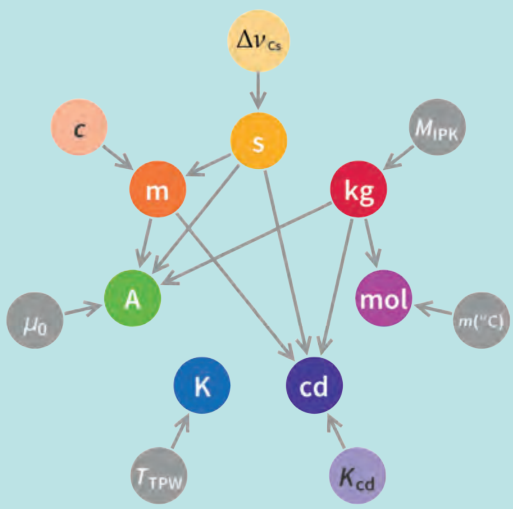


Fig (b) Old SI

Exercises

1. Choose the correct option.

i) $[L^1M^1T^{-2}]$ is the dimensional formula for
(A) Velocity (B) Acceleration
(C) Force (D) Work

ii) The error in the measurement of the sides of a rectangle is 1%. The error in the measurement of its area is
(A) 1% (B) 1/2%
(C) 2% (D) None of the above.

iii) Light year is a unit of
(A) Time (B) Mass
(C) Distance (D) Luminosity

iv) Dimensions of kinetic energy are the same as that of
(A) Force (B) Acceleration
(C) Work (D) Pressure

v) Which of the following is not a fundamental unit?
(A) cm (B) kg
(C) centigrade (D) volt

2. Answer the following questions.

i) Star A is farther than star B. Which star will have a large parallax angle?

ii) What are the dimensions of the quantity $l\sqrt{l/g}$, l being the length and g the acceleration due to gravity?

iii) Define absolute error, mean absolute error, relative error and percentage error.

iv) Describe what is meant by significant figures and order of magnitude.

v) If the measured values of two quantities are A ± ΔA and B ± ΔB, ΔA and ΔB being the mean absolute errors. What is the maximum possible error in A ± B? Show that if $Z = \frac{A}{B}$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

vi) Derive the formula for kinetic energy of a particle having mass *m* and velocity v using dimensional analysis

3. Solve numarical examples.

i) The masses of two bodies are measured to be 15.7 ± 0.2 kg and 27.3 ± 0.3 kg. What is the total mass of the two and the error in it?

[Ans : 43 kg, ± 0.5 kg]

ii) The distance travelled by an object in time (100 ± 1) s is (5.2 ± 0.1) m. What is the speed and it's relative error?

[Ans : $0.052\ ms^{-1}$, $\pm 0.0292\ ms^{-1}$]

iii) An electron with charge *e* enters a uniform. magnetic field $\vec{B}$ with a velocity $\vec{v}$. The velocity is perpendicular to the magnetic field. The force on the charge *e* is given by

$|\vec{F}| = Bev$ Obtain the dimensions of $\vec{B}$.

[Ans: $[L^0M^1T^{-2}I^{-1}]$]

iv) A large ball 2 m in radius is made up of a rope of square cross section with edge length 4 mm. Neglecting the air gaps in the ball, what is the total length of the rope to the nearest order of magnitude?

[Ans : $\approx 10^6$ m = 10^3km]

v) Nuclear radius *R* has *a* dependence on the mass number (*A*) as $R = 1.3\times10^{-16}A^{1/3}$ m. For a nucleus of mass number *A*=125, obtain the order of magnitude of *R* expressed in metre.

[Ans : -15]

vi) In a workshop a worker measures the length of a steel plate with a Vernier callipers having a least count 0.01 cm. Four such measurements of the length yielded the following values: 3.11 cm, 3.13 cm, 3.14 cm, 3.14 cm. Find the mean length, the mean absolute error and the percentage error in the measured value of the length.

[Ans: 3.13 cm, 0.01 cm, 0.32%]

vii) Find the percentage error in kinetic energy of a body having mass 60.0 ± 0.3 g moving with a velocity 25.0 ± 0.1 cm/s.

[Ans: 1.3%]

viii) In Ohm's experiments , the values of the unknown resistances were found to be 6.12 Ω , 6.09 Ω, 6.22 Ω, 6.15 Ω. Calculate the mean absolute error, relative error and percentage error in these measurements.

[Ans: 0.04 Ω ,0.0065 Ω , 0.65%]

ix) An object is falling freely under the gravitational force. Its velocity after travelling a distance h is v. If v depends upon gravitational acceleration g and distance, prove with dimensional analysis that $v = k\sqrt{gh}$ where k is a constant.

x) $v = at + \frac{b}{t+c} + v_0$ is a dimensionally valid equation. Obtain the dimensional formula for a, b and c where v is velocity, t is time and v_0 is initial velocity.

[Ans: a- $[L^1M^\circ T^{-2}]$, b- $[L^1M^\circ T^\circ]$, c- $[L^\circ M^\circ T^1]$]

xi) The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

[Ans: 4.255 m^2, 8.552 m^3]

xii) If the length of a cylinder is l = (4.00±0.001) cm, radius r = (0.0250 ±0.001) cm and mass m = (6.25±0.01) gm. Calculate the percentage error in the determination of density.

[Ans: 8.185%]

xiii) When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72'' of arc. Calculate the diameter of the Jupiter.

[Ans: 1.428×10^5 km]

xiv) If the formula for a physical quantity is $X = \frac{a^4 b^3}{c^{1/3} d^{1/2}}$ and if the percentage error in the measurements of a, b, c and d are 2%, 3%, 3% and 4% respectively. Calculate percentage error in X.

[Ans: 20%]

xv) Write down the number of significant figures in the following: 0.003 m^2, 0.1250 gm cm^{-2}, 6.4 x 10^6 m, 1.6 x 10^{-19} C, 9.1 x 10^{-31} kg.

[Ans: 1, 4, 2, 2, 2]

xvi) The diameter of a sphere is 2.14 cm. Calculate the volume of the sphere to the correct number of significant figures.

[Ans: 5.13 cm^3]

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